Online-Appendix zu

„Extending Kolkata Paise Restaurant Problem to Dynamic Matching in Mobility Markets“

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A. KPRP: Limited Learning 2

This strategy was also presented in the original paper [1].

Agents start with a random choice of restaurants the first day.

- If some agent gets dinner at restaurant j at time t, he chooses randomly from those restaurants $j' \in J$ with $u(j') > u(j)$ at time $t+1$ (*improvement*).
- Otherwise, he chooses randomly from those restaurants $j' \in J$ with $u(j') < u(j)$ at time $t+1$ (*worsening*).

Chakrabarti et al. give no formula by which the average utilization fraction is calculated or simulation results, only a value of $f = 0.5$. They argue that it should not make a difference if the agent improves after getting dinner or after not getting dinner (and worsening vice versa) and therefore conclude that the utilization should always be 50%.

In my simulations, I was not able to reproduce this result, I found an average utilization of $f \approx 0.434$ for $N = 1000$ agents and 10^6 time steps. Simulating with less time steps or less agents both returned higher utilization fractions, which could explain the result from [1].

One of the issues with this strategy is the imbalance of between the different restaurants: High ranking restaurants are often missed during the improvement step (red), very low ranking restaurants are often missed during the worsening step (green). Mediocre restaurants are relatively seldomly missed. This result can also be seen in figure A.1 which depicts the number of missed dinners per restaurant (during improvement, during worsening, and in total (black)) on a logarithmic scale for $N = 50$ restaurants.

The correlation between the number of restaurants and the number of misses assuming 10,000 time steps is shown in figure A.2. For this experiment, I simulate 10^1 to 10^5 agents and restaurants (x-axis).

The argument given in the paper that it should not make a difference which group improves and which worsens (those who got dinner and those who did not) does not hold, as it might happen that a restaurant is frequented by exactly one agent, who

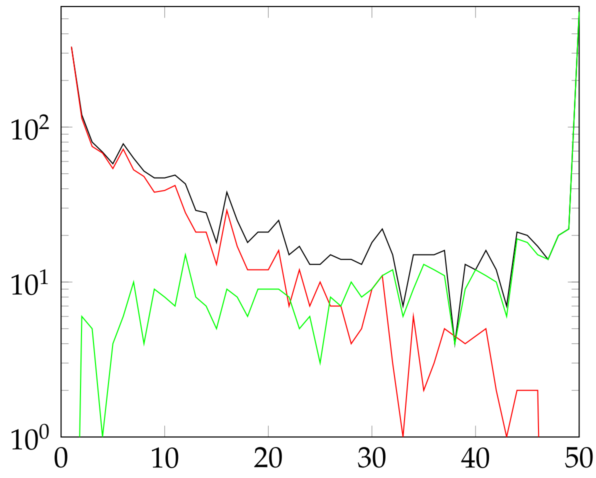

Figure A.1.: Missed Dinners per Restaurant

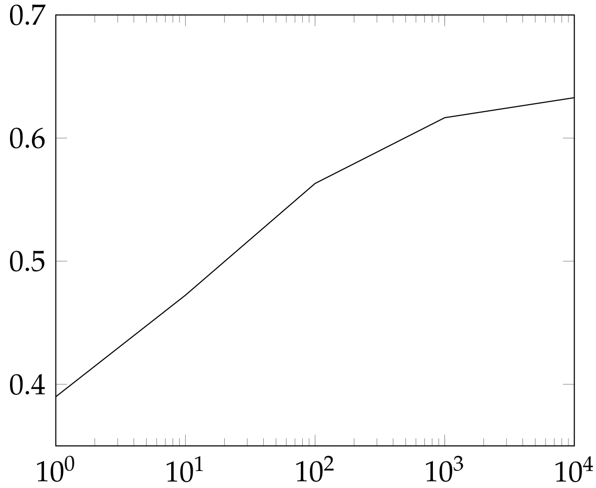

Figure A.2.: Missed Dinners Compared to Number of Restaurants improves (without anyone worsening) or more than two agents (one improving and the other ones worsening). The argument that the number of people worsening and improving should be identical only holds if every restaurant is frequented by exactly two agents (or no agent at all).

B. MP model: Influence of Mixing Ratio on Number of First Preference Customers

From equation 6.10 we expect approximately 77 distinct customers with highest utility for $N = 1000$. In simulations, this value varied from 69 to 83. The average probability of the #1 priority of the shared preference ranking of being the #1 priority of some agent is $\approx 4.0\%$ (in comparison to 0.1% in the *individual preferences* case). Between the #100 priority and #1 priority the probability grows overlinearily, the probability for values around the #35 priority to be among the top priorities is still increased by a factor of 10, whilst the probability that a customer which is 70^{th} in the shared ranking to be an individual top priority is approximately even to the probability of being the top priority in the *individual preferences* case.

Figure B.1 depicts the influence of the mixing ratio between shared and individual component on the number of distinct first preferences. If the individual component is being neglected (ratio 0.0), there is exactly 1 first preference. If the shared component is being neglected (ratio 1.0), there are approximately 632 different first preferences ($N \cdot \left(1 - e^{-1}\right)$).

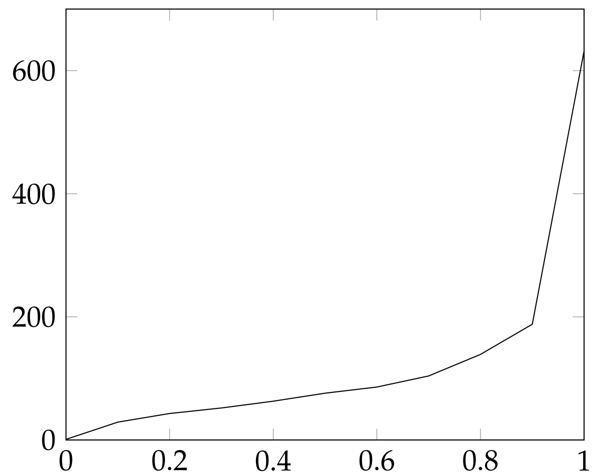

Figure B.1.: Comparing the influence of mixing ratio between shared and individual component for the Strict Rank Dependent Choice Strategy

C. IPMC model: Type of Selection

C.1. No Learning: Selecting Districts

To calculate the number of missed customers or the number of agents who are not assigned a customer (and therefore the utilization fraction), we have to distinguish by the number of customers in a district. In theory, there might be $0 \ldots N$ customers in one district, though both extremes will get highly unlikely. Assuming that there are 5 customers on average per district, the probability $C(c_k)$ for capacity c_k is given by equation C.2. In this case φ is the average number of customers per district (in simulations: 5).

$$C(c_k) = \binom{\varphi N}{c_k} \frac{1}{N_k}^{c} \left(1 - \frac{1}{N}\right)^{\varphi N - c_k} \tag{C.1}$$

$$= \frac{\varphi^{c_k}}{c_k!} e^{-\varphi} \tag{C.2}$$

For every district, we now calculate the number of customers which are not being served based on the capacity.

$$O(o_k, c_k) = \begin{cases} \frac{\varphi^{o_k}}{o_k!} e^{-\varphi}, & \text{if } c_k > o_k \\ 0 & \text{otherwise} \end{cases} \tag{C.3}$$

We now connect equations C.2 and C.3 to equation C.5.

$$f = \frac{1}{\varphi} \sum_{c_k=1}^{N} C(c_k) \cdot \left(c_k - \sum_{o_k=0}^{c_k-1} O(o_k, c_k) \cdot (c_k - o_k) \right) \tag{C.4}$$

$$= \frac{1}{\varphi} \sum_{c_k=1}^{N} \frac{\varphi^{c_k}}{c_k!} e^{-\varphi} \cdot \left(c_k - \sum_{o_k=0}^{c_k-1} \frac{\varphi^{o_k}}{o_k!} e^{-\varphi} \cdot (c_k - o_k) \right) \tag{C.5}$$

For $\varphi = 5$ this results in an average utilization fraction of $f = 75.09\%$. The utility is given by average utility u_{avg} for every agent who gets dinner, thus $u = 0.379 \cdot u_{max}$ for $N = 1000$ agents.

It shall be mentioned that setting $\varphi = 1$ does not reduce the problem to the *No Learning* strategy in the *Kolkata Paise Restaurant Problem*, but yields $f = 47.62\%$, as some districts will contain no customers at all.

C.2. Rank Dependent Choice: Selecting the District with the Highest Average Utility

In this sub-strategy, every agent chooses the district with the highest average utility.

In simulations, this strategy preferred districts with very few available customers over larger districts (as there is a smaller probability of only having high values).

In simulations with $N = 100$ agents and $\varphi = 5$ customers per district, the utilization was as low as $\bar{f} = 46\%$, as one district with only one customer in it was preferred by 25 agents.

Thus, this strategy is not recommended for $N \to \infty$.

C.3. Switch if Customer is Overcrowded

If there is more than one agent choosing the same customer, all of them will return with probability $\frac{1}{o_k}$ and choose any other (randomly drawn) customer with probability $\frac{o_k - 1}{o_k}$. Thus, agents might be appealed to choose another district even if their district is not overcrowded.

The utilization fraction is $f = 88.9\%$, which only slightly improves in comparison to the basic *Kolkata Paise Restaurant Problem*. The utility is given by $u = fN = 44.5\%$ for $N = 1000$.