Online-Appendix zu

„ Analysis of Green Bonds“

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Junior Management Science 7(3) (2022) 668-689

A Appendix

A.1 Derivation of Trinomial Tree Model

We derive the probabilities for the trinomial tree following the assumptions in **?**. For the default branching method (i.e., $j_{min} < j < j_{max}$), the condition for the expected change notates as,

$$\begin{aligned} p_u \cdot \Delta s + p_m \cdot 0 + p_d \cdot (-\Delta s) &= \mathbb{E}[dLP_t] \\ &= -a \cdot j \cdot \Delta s \cdot \Delta t \end{aligned} \tag{26}$$

Dividing by Δs and solving for p_d yields

$$p_d = p_u + a \cdot j \cdot \Delta t. \tag{27}$$

We use this result in the condition for the variance

$$\begin{aligned} p_u \cdot \Delta s^2 + p_m \cdot 0^2 + p_d \cdot \Delta s^2 &= \sigma^2 \cdot \Delta t + a^2 \cdot j^2 \cdot \Delta s^2 \cdot \Delta t^2 \\ p_u \cdot \Delta s^2 + (p_u + a \cdot j \cdot \Delta t) \cdot \Delta s^2 &= \sigma^2 \cdot \Delta t + a^2 \cdot j^2 \cdot \Delta s^2 \cdot \Delta t^2 \end{aligned} \tag{28}$$

and solve for p_u to obtain

$$p_u = \frac{1}{2}\sigma^2 \cdot \frac{\Delta t}{\Delta s^2} + \frac{1}{2}a^2 \cdot j^2 \cdot \Delta t^2 - \frac{1}{2}a \cdot j \cdot \Delta t. \tag{29}$$

We use $\Delta s = \sigma\sqrt{3\Delta t}$ or $\Delta s^2 = \sigma^2 \cdot 3\Delta t$ to obtain

$$\begin{aligned} p_u &= \frac{1}{2}\sigma^2 \cdot \frac{\Delta t}{\sigma^2 \cdot 3\Delta t} + \frac{1}{2}\left(a^2 \cdot j^2 \cdot \Delta t^2 - a \cdot j \cdot \Delta t\right) \\ &= \frac{1}{6} + \frac{1}{2}\left(a^2 \cdot j^2 \cdot \Delta t^2 - a \cdot j \cdot \Delta t\right). \end{aligned} \tag{30}$$

We use this result to obtain the probability p_d as

$$\begin{aligned} p_d &= p_u + a \cdot j \cdot \Delta t \\ &= \frac{1}{6} + \frac{1}{2}\left(a^2 \cdot j^2 \cdot \Delta t^2 + a \cdot j \cdot \Delta t\right). \end{aligned} \tag{31}$$

Finally, we use $p_u + p_m + p_d = 1$ to obtain

$$\begin{aligned} p_m &= 1 - p_u - p_d \\ &= \frac{2}{3} - a^2 \cdot j^2 \cdot \Delta t^2. \end{aligned} \tag{32}$$

At the limits of the trinomial tree (i.e., j_{min} and j_{max}), the branching structure changes as displayed in Figure 5.

First, we compute the probabilities for the lower limit j_{min}. For this, we change the condition for the expected change to,

$$\begin{aligned} p_u \cdot 2\Delta s + p_m \cdot \Delta s + p_d \cdot 0 &= \mathbb{E}[dLP_t] \\ &= -a \cdot j \cdot \Delta s \cdot \Delta t, \end{aligned} \tag{33}$$

as the liquidity premium cannot decrease any further. We obtain

$$p_m = -2p_u - a \cdot j \cdot \Delta t, \tag{34}$$

which we substitute into the new condition for the variance

$$\begin{aligned} p_u \cdot 4\Delta s^2 + p_m \cdot \Delta s^2 + p_d \cdot 0^2 &= \sigma^2 \cdot \Delta t + a^2 \cdot j^2 \cdot \Delta s^2 \cdot \Delta t^2 \\ p_u \cdot 4\Delta s^2 + (-2p_u - a \cdot j \cdot \Delta t) \cdot \Delta s^2 + p_d \cdot 0^2 &= \sigma^2 \cdot \Delta t + a^2 \cdot j^2 \cdot \Delta s^2 \cdot \Delta t^2. \end{aligned} \tag{35}$$

We solve the equation for p_u and use $\Delta s^2 = \sigma^2 \cdot 3\Delta t$ to obtain

$$\begin{aligned} p_u &= \frac{1}{2}\sigma^2 \cdot \frac{\Delta t}{\sigma^2 \cdot 3\Delta t} + \frac{1}{2}\left(a^2 \cdot j^2 \cdot \Delta t^2 + a \cdot j \cdot \Delta t\right) \\ &= \frac{1}{6} + \frac{1}{2}\left(a^2 \cdot j^2 \cdot \Delta t^2 + a \cdot j \cdot \Delta t\right). \end{aligned} \tag{36}$$

We use this result to obtain the probability p_m as

$$\begin{aligned} p_m &= -2p_u - a \cdot j \cdot \Delta t \\ &= -\frac{1}{3} - a^2 \cdot j^2 \cdot \Delta t^2 - a \cdot j \cdot \Delta t - a \cdot j \cdot \Delta t \\ &= -\frac{1}{3} - a^2 \cdot j^2 \cdot \Delta t^2 - 2a \cdot j \cdot \Delta t. \end{aligned} \tag{37}$$

Finally, we use $p_u + p_m + p_d = 1$ to obtain

$$\begin{aligned} p_d &= 1 - p_u - p_m \\ &= \frac{7}{6} + \frac{1}{2}\left(a^2 \cdot j^2 \cdot \Delta t^2 + 3a \cdot j \cdot \Delta t\right). \end{aligned} \tag{38}$$

In the same fashion, we can compute the probabilities for j_{max}. In this case, we change the condition for the expected change to,

$$p_u \cdot 0 + p_m \cdot (-\Delta s) + p_d \cdot (-2\Delta s) = \mathbb{E}[dLP_t] \tag{39}$$

and the condition for the variance to

$$p_u \cdot 0 + p_m \cdot \Delta s^2 + p_d \cdot 4\Delta s^2 = \sigma^2 \cdot \Delta t + a^2 \cdot j^2 \cdot \Delta s^2 \cdot \Delta t^2. \tag{40}$$

In this case, we obtain

$$\begin{aligned} p_u &= \frac{7}{6} + \frac{1}{2}\left(a^2 \cdot j^2 \cdot \Delta t^2 - 3a \cdot j \cdot \Delta t\right) \\ p_m &= -\frac{1}{3} - a^2 \cdot j^2 \cdot \Delta t^2 + 2a \cdot j \cdot \Delta t \\ p_d &= \frac{1}{6} + \frac{1}{2}\left(a^2 \cdot j^2 \cdot \Delta t^2 - a \cdot j \cdot \Delta t\right) \end{aligned} \tag{41}$$

Based on the calibration result of the model in section 4.3, we use these formulas to compute the probabilities for the trinomial tree. The probabilities are displayed in Table 8.

Table 8: Trinomial tree probabilities

	j	p_u	p_m	p_d
j_{max}	4	0.9008	0.0093	0.0900
	3	0.1058	0.6465	0.2477
	2	0.1238	0.6577	0.2184
	1	0.1441	0.6644	0.1914
	0	0.1667	0.6667	0.1667
	-1	0.1914	0.6644	0.1441
	-2	0.2184	0.6577	0.1238
	-3	0.2477	0.6465	0.1058
j_{min}	-4	0.0900	0.0093	0.9008

The table shows the probabilities for the trinominal tree using $\sigma = 0.0031$, $a = 11.9$, $T = 3.1$ years, $N = 791$ and assuming $\Delta s = \sigma\sqrt{3\Delta t}$.

A.2 Additional Robustness Tests for Switch Option Value

Table 9: Option value at execution for different interest rate r

r	LP	$ST^{\max}$
-100	12.06	4.06
0	12.06	4.06
100	12.06	4.06
200	12.06	4.06

The table shows the values of $ST^{\max}$ for different r based on $GP = 8$bp, $\sigma = 0.0031$, $a = 11.9$, $T = 3.1$ years and a trinomial tree length of $N = 791$.

Table 10: Option value at execution for different rounding precision

Precision	LP	$ST^{\max}$
4	8	0.35
5	10.31	2.36
6	12.06	4.06
7	12.35	4.35
8	12.35	4.35

The table shows the values of $ST^{\max}$ for different rounding precisions, measured in digits after the decimal point. A precision of six digits equals 0.01bp. The further parameters are $GP = 8$bp, a risk-free rate of $r_f = 200$bp, $\sigma = 0.0031$, $a = 11.9$, $T = 3.1$ years and a trinomial tree length of $N = 791$.

A.3 Additional Estimation Results and Statistics

Table 11: Summary statistics for BAS of German twin bonds

	Mean	SD	Min	p25	Median	p75	Max	N
BAS_{G2050}	0.67	0.31	0.00	0.50	0.60	0.90	1.50	131
BAS_{C2050}	0.57	0.27	0.10	0.40	0.50	0.90	1.00	131
BAS_{G2031}	0.57	0.33	0.20	0.30	0.50	0.90	1.30	47
BAS_{C2031}	0.29	0.11	0.20	0.20	0.30	0.30	0.80	47
BAS_{G2030}	0.70	0.36	0.20	0.40	0.60	0.90	1.70	305
BAS_{C2030}	0.29	0.24	0.10	0.20	0.20	0.30	1.70	305
BAS_{G2025}	1.27	0.85	0.20	0.60	1.15	1.40	3.30	262
BAS_{B2025}	0.57	0.26	0.30	0.40	0.50	0.70	2.40	262

The table shows the summary statistics (i.e., mean, standard deveation (SD), minimum, 25th percentile (p25), median, 75th percentile (p75), maximum and number of observations (N)) for the bid-ask spread of the closing yields in basis points of the German twin bonds displayed in Table 1. The data is retrieved from Refinitiv Eikon (Accessed: 10.11.2021) and covers the period from 09.09.2020 to 10.11.2021.

Table 12: Summary statistics for yield of German twin bonds

	Mean	SD	Min	p25	Median	p75	Max	N
y_{G2050}	15.96	13.95	-9.80	4.70	17.00	26.70	41.40	127
y_{C2050}	6.12	17.73	-48.60	-8.00	4.30	20.60	44.80	566
y_{G2031}	-25.34	8.40	-40.70	-35.10	-23.70	-17.90	-13.50	43
y_{C2031}	-29.71	12.59	-49.80	-41.90	-30.10	-18.80	-9.70	103
y_{G2030}	-47.15	12.86	-66.90	-59.10	-48.50	-35.40	-21.20	301
y_{C2030}	-43.02	12.69	-64.10	-54.30	-45.50	-31.80	-15.70	358
y_{G2025}	-72.22	6.80	-86.70	-76.80	-72.20	-67.40	-54.50	258
y_{C2025}	-69.38	7.02	-83.70	-74.55	-69.50	-64.65	-46.60	344

The table shows the summary statistics for the yields in basis points of the German twin bonds displayed in Table 1. The data is retrieved from Refinitiv Eikon (Accessed: 04.11.2021).

Table 13: Summary statistics for Δy between German twin bonds

	Mean	SD	Min	p25	Median	p75	Max	N
Δy_{2050}	-3.95	0.45	-5.70	-4.30	-4.00	-3.60	-3.10	126
Δy_{2031}	-4.41	0.42	-5.50	-4.70	-4.40	-4.20	-3.20	43
Δy_{2030}	-4.76	1.64	-7.60	-6.30	-5.10	-3.30	-1.40	301
Δy_{2025}	-3.36	1.88	-8.50	-4.40	-3.10	-2.30	0.10	258

The table shows the summary statistics for the yield spread (i.e., $\Delta y = y_G - y_C$) in basis points between the German twin bonds displayed in Table 1. The data is retrieved from Refinitiv Eikon (Accessed: 04.11.2021).

Table 14: Summary statistics for liquidity proxy

	Mean	SD	Min	p25	Median	p75	Max	N
LP_{2050}	93.08	9.61	50.54	89.01	93.66	98.55	116.80	549
LP_{2031}	48.76	6.47	36.26	43.77	47.51	52.58	80.15	549
LP_{2030}	48.61	6.31	36.21	43.95	47.55	51.92	78.25	549
LP_{2025}	48.66	6.44	38.01	44.23	47.49	50.66	72.24	549

The table shows the summary statistics for the estimated proxy of the liquidity premium in basis points. The data is based on published yield curves by the **?** and covers the period from 02.09.2019 until 01.11.2021.

Table 15: Summary of OLS estimation results for Vasicek process

	2050	2031	2030	2025
LP_{t-1}	0.7662***	0.9023***	0.9078***	0.9534***
	(0.0537)	(0.0231)	(0.0223)	(0.0159)
Constant	0.0022***	$0.0005 * **$	0.0004***	0.0002***
	(0.0005)	(0.0001)	(0.0001)	(0.0001)
R-squared	0.5863	0.8150	0.8250	0.9089
N	548	548	548	548

Robust standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

The table shows the estimation results for the process of the liquidity premium specified in Equation 22. The dependent variable is the daily measured liquidity procxy LP_t and the independent variables are a constant and the lagged value LP_{t-1}. The data is based on published yield curves by the **?** and covers the period from 02.09.2019 until 01.11.2021.

A.4 List of German Twin Bonds

Table 16: German Twin Federal Securities

Name	First Issue Date	Last Issuance	Maturity Date	Coupon	Outstanding	Type	ISIN	RIC
2021 (2050) Bund/g	18.5.2021	11.05.2021	15.8.2050	0.00%	6.0 bn. EUR	Green	DE0001030724	DE103072=
2019 (2050) Bund	23.8.2019	18.08.2021	15.8.2050	0.00%	29.0 bn. EUR	Brown	DE0001102481	DE110248=
2021 (2031) Bund/g	10.9.2021	20.10.2021	15.8.2031	0.00%	6.5 bn. EUR	Green	DE0001030732	DE103073=
2021 (2031) II Bund	18.6.2021	10.11.2021	15.8.2031	0.00%	26.5 bn. EUR	Brown	DE0001102564	DE110256=
2020 (2030) Bund/g	9.9.2020	02.09.2020	15.8.2030	0.00%	6.5 bn. EUR	Green	DE0001030708	DE103070=
2020 (2030) II Bund	19.6.2020	18.11.2020	15.8.2030	0.00%	30.5 bn. EUR	Brown	DE0001102507	DE110250=
Bobl/g	6.11.2020	04.11.2020	10.10.2025	0.00%	5.0 bn. EUR	Green	DE0001030716	DE103071=
Bobl	10.7.2020	02.12.2020	10.10.2025	0.00%	25.0 bn. EUR	Brown	DE0001141828	DE114182=

Source: **?** and Refinitiv Eikon (Accessed: 21.11.2021)