

" Carbon Pricing: A Comparison between Germany and the United Kingdom"

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A Appendix

Dynamic Conditional Correlation Multivariate Generalized Autoregressive Conditional Heteroskedasticity Model

There is another model that could have served to test the two hypotheses of this thesis. Unfortunately, it could not be used reliably. The residuals were not normally distributed and presented heteroskedasticity in their variance. Still, it is useful to share it for research purposes. Finally, it is important to mention that the DCC-MGARCH model has been used in literature, but mostly to explain financial variables, which are volatile. The model will be specified in the next paragraphs.

This paper has two main objectives: first, to show whether the UK CPF has been more effective in tackling the CO_2 emissions in the UK than the EU ETS in Germany. A coefficient between the carbon price and the CO_2 emissions will be calculated to determine the magnitude of this relationship in each country. Second, test whether there are spillovers effects between the carbon price and electricity produced by renewables in these two countries. The spillover effect will be calculated by the same econometric model. The magnitude of the spillover effect is important since the goal of Germany and the UK is to produce clean electricity in the long term. Therefore, the carbon price must promote the elimination of GHG emissions and not the switch from coal-fired plants to gas. Finally, the seasonality effect is being considered.

The weekly logarithmic variation of energy and economic variables are used. This approach goes in line with (Manera, Nicolini, & Vignati, 2013), who employed the same econometric model to test correlations among energy commodities. The energy variables used price of natural gas, coal, EU ETS and UK CPF, the electricity demand, the electricity production from solar, wind, and nuclear sources; and CO_2 emissions of Coal, Natural Gas, and Lignite. The Coal-to-Gas price ratio has been used by (Gugler et al., 2021), (Abrell et al., 2021), and many others because it represents the cost relationship between the two most important electricity fuels. (Gugler et al., 2021) and (Koch et al., 2014) utilized the production from renewable sources in their models. The electricity production from renewables influences the ones from coal and natural gas because the are ranked in the merit-order curve. The economic variables used are the prices of the Financial Times Stock Exchange 100 (FTSE 100), which represents the 100 biggest companies listed in the London Stock Exchange, and the *Deutscher Aktien Index* (DAX), which represents the 30 largest companies listed in the Frankfurt Stock Exchange. Several authors have included

economic variables in their analysis of the carbon price. For example, (Koch et al., 2014) employed the returns of the European stock exchange and concluded that the EU ETS was not affected by demand shocks. Still, there is not a homogeneous consensus of the effects of an economic recession on the carbon price. During the time frame analyzed in this document, the Covid-19 economic crisis took place.

As the carbon price is time-varying, it is crucial for its correct modeling to 1) measure the volatility of the electricity production per source and 2) consider the economic variables that have an impact on their price. For the first point, the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model is useful. It assumes that not only past variations but also the volatility of the CO_2 emissions has an impact on the present value of the CO_2 emissions. For the second point, the GARCH multivariate Dynamic Conditional Correlation model (DCC-GARCH) considers that past variations and volatility of the other variables employed, such as the carbon price, affect the CO_2 emissions. The added value of the multivariate DCC-GARCH model is that both points can be tested per variable. Hence, the model will measure the impact that previous returns and deviations of each variable have on itself, and how these affect the other variables dynamically.

The univariate GARCH model specified by Engle (2013) is as follows:

(i)
$$r_t = m_t + \sqrt{h_t} \varepsilon_t$$

(ii)
$$h_{t+1} = \omega + \alpha (r_t - m_t)^2 + \beta h_t = \omega + \alpha h_t \varepsilon_t^2 + \beta h_t$$

The first equation (i) represents the returns of the financial asset (r_t) in terms of the average of the returns (m_t) and the residuals $(\sqrt{h_t}\varepsilon_t)$. h_t represents the variance of the residuals, and ε_t the error term, which has a variance of one. The second equation (ii) is the GARCH model for the variance of the residuals (h_{t+1}) which is explained by the past realizations of itself (h_t) and the constants ω , α , and β . To estimate the constants ω , α , and β ; the model updates the previous forecast of h and the residual (Engle, 2013). The way the model optimizes is considering weights of $(1 - \alpha - \beta, \alpha, \beta)$. Note that the weight of ω is calculated by difference the $(1 - \alpha - \beta)$.

In this paper, the Dynamic Conditional Correlation Multivariate GARCH (MGARCH-DCC) is applied, which allows the use of many variables without increasing the model's complexity. The MGARCH-DCC model is used in this paper for the following reasons: 1) the model captures the heteroscedasticity of the variance, and 2) estimates the conditional correlations between electricity variables that vary over time (Bali and Engle (2010); Ewing, Malik, and Ozfidan (2002)). For instance, the correlation between the carbon price and CO_2 emissions may decrease during an economic crisis, because demand for electricity

generated by coal may shrink due to economic depression instead. The MGARCH-DCC model is determined in two steps. First, each variable is estimated following a univariate GARCH model. Second, the correlation among all the variables is calculated. This dynamic process is expressed by the variable $\rho_{ij,t}$ that alters the MGARCH model as shown below.

(iii)
$$h_{ij,t} = \rho_{ij,t} \sqrt{h_{ii,t} h_{jj,t}}$$

The variable $\rho_{ij,t}$ represents the dynamic conditional correlation among all the variables included in the MGARCH. The i represents the number of variables considered in the model. For this paper, there are six variables per country: clean electricity generation, CO_2 emissions from coal, gas and lignite, carbon price, electricity demand, coal-to-gas price ratio and the stock market index. The j represents the same as i, but the distinction is made in order to emphasize that the model is expressed in matrix terms and that different combinations of the variables are considered. The t represents the day of the observations since the MGARCH is a time-series model.

Therefore, the MGARCH-DCC specified by Engle (2002) is comprised as follows:

(iv)
$$X_t = \mu_t + H_t^{1/2} \varepsilon_t$$

(v)
$$H_t = D_t R_t D_t$$

(vi)
$$R_t = Q_t^{*-1} Q_t Q_t^{*-1}$$

(vii)
$$D_t = diag(\sqrt{h_{11,t}}, \dots, \sqrt{h_{jj,t}})$$

(viii)
$$Q_{t+1} = (1 - \alpha - \beta)$$
\$ = $Q + \alpha Q_t + \beta \delta_{i,t} \delta_{j,t}$

The fourth equation (iv) represents the univariate GARCH model with the inclusion of the matrix of time-varying conditional covariance $(H_t^{1/2})$. This equation (iv) expresses the vector of returns (X_t) and the vector of conditional returns (μ_t) per asset, where ε_t is the vector of standardized residuals. H_t which is represented by the fifth equation (v) is a matrix of time-varying conditional covariance that is built by the matrices D_t and R_t . The diagonal matrix D_t which is shown in the seventh equation (vii) consists of the standard deviations of the estimated returns by the univariate GARCH model.

$$D_{t} = \begin{pmatrix} \sigma_{1,t}^{2} & 0 & \cdots & 0 \\ 0 & \sigma_{2,t}^{2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{i,t}^{2} \end{pmatrix}$$

The eighth equation (viii) shows Q_{t+1} which represents the unconditional correlation matrices that build the R_t , which is a symmetric conditional correlation matrix. This

equation (viii) exhibits the coefficients that are relevant to test our hypotheses. R_t is represented by the sixth equation (vi). Finally, the model obliges that α and β are non-negative and that the sum of both is lower than 1.

$$R_{t} = \begin{pmatrix} 1 & \rho_{12,t} & \cdots & \rho_{1i,t} \\ \rho_{12,t} & 1 & \cdots & \rho_{2i,t} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1i,t} & \rho_{2i,t} & \cdots & 1 \end{pmatrix}$$

The alpha (α) represents the spillover effect, which shows if the variation in the volatility of a carbon price impact another. For example, if there is an abrupt change in the price of price of the EUA or UK CPF, the α will measure how this increase in volatility impacts the CO_2 emissions. The beta (β) outlines the persistent effects that exhibit if this variation has a future repercussion in another asset. For instance, if there is an economic crisis, the volatility of carbon price would increase positively. This could lead to a increase in the volatility of CO_2 emissions that would last for many periods ahead, as the carbon price is considered a cost for the generation of electricity by fossil fuels. These two variables are useful to test if the carbon's volatility affects the volatility of the CO_2 emissions and if the impact is persistent enough to influence persistently the future development of the CO_2 emissions. These effects are important for the industry, because clean electricity investments require a stable carbon price to be profitable (Hirst, 2018). Under the multivariate model, the α is represented by the variable dcca and the β by dccb and they test the effect among the six variables.

Q-Q Plots for Residuals of the DCC-MGARCH model

As mentioned above, the model was not used since its residuals are not normally distributed. Below are the Q-Q plots which show this behaviour.

Germany: Normal Q-Q Plot using DCC-MGARCH

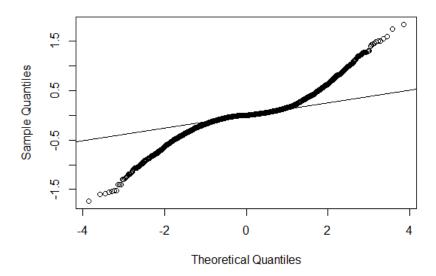


Figure created by the author

UK: Normal Q-Q Plot using DCC-MGARCH

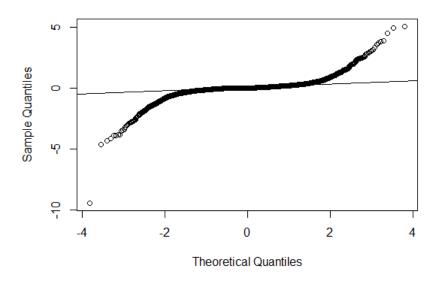


Figure created by the author

Q-Q Plots of Residuals

The Q-Q Plots of the models used in this study are shown below. As mentioned above, the residuals of the models employed are normally distributed.

UK: Normal Q-Q Plot for CO2 from Coal using OLS

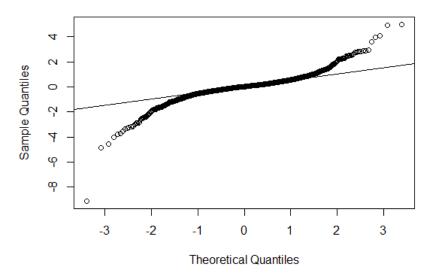


Figure created by the author

UK: Normal Q-Q Plot for CO2 from Gas using OLS

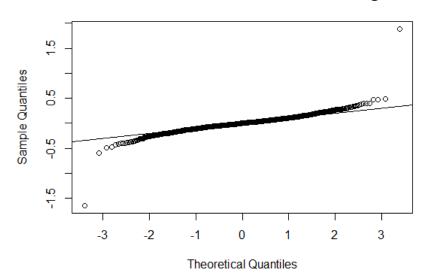


Figure created by the author

Germany: Normal Q-Q Plot for CO2 from Coal using OLS

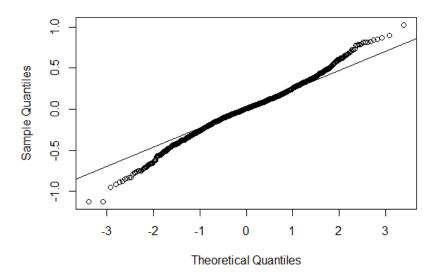


Figure created by the author

Germany: Normal Q-Q Plot for CO2 from Gas using OLS

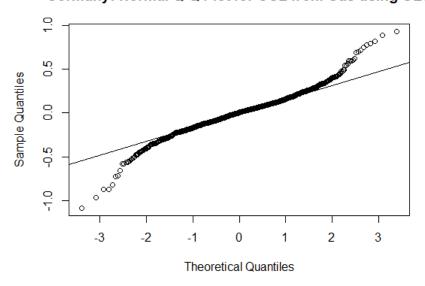


Figure created by the author

Germany: Normal Q-Q Plot for CO2 from Lignite using OLS

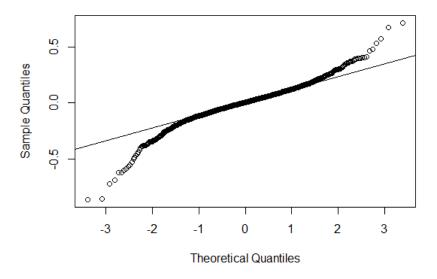


Figure created by the author

Normal Q-Q Plot for Differences in Differences using OLS

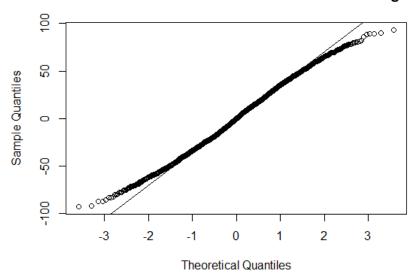


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